A constant shear stress immediately above the HEL requires $d\sigma_\tau/dV = 0$. In this case the shock compressibility and isotropic compressibility are equal and a disturbance will travel with the bulk sound speed. On the other hand a significant change in $\sigma_\tau$ requires $d\sigma_\tau/dV \neq 0$ and a change in shock compressibility will result. If the shear strength is reduced the shock compressibility will be less than the isotropic compressibility and a disturbance will travel with a velocity less than bulk sound speed. In the absence of a phase transition this 'slow second-wave' is an explicit indication of large decreases in shear strength.

One of the experiments was conducted at a pressure just above the HEL. The second-wave shock velocity on experiment 178-66 is calculated to 6·0 mm/$\mu$sec compared to the bulk sound speed of 8·0 mm/$\mu$sec. Two other experiments (192-64 and 357-67) also showed shock velocities substantially less than bulk sound speed. The method of data reduction is admittedly subject to error because of the interaction of the first and second waves. However, the reduced data in experiment 178 cannot be changed sufficiently by various assumptions in the data analysis to obtain a shock velocity as large as the bulk sound speed. Thus, the slow second wave velocity data give a positive indication of the loss of shear strength for pressures just above the HEL.

Wackerle's shock compression study of crystalline quartz[24] also showed one experiment in which the shock velocity was substantially less than bulk sound speed. Although Wackerle did not comment on this observation, it is now evident that both sapphire and quartz show similarities in their compressional behaviors in the immediate vicinity of the HEL. Recent piezoelectric response measurements confirm the slow second wave behavior for quartz[38].

(e) *Determination of shear offset*

Although it is not common to do so, the high pressure shock data can be used to give an explicit measure of the shear offset. Under the assumption that all high pressure data points have a constant offset from the isotropic compression curve and that no phase changes have occurred, the high pressure data can be extrapolated to zero stress and the observed offset in relative volume between the extrapolation and the initial relative volume can be used to compute the shear stress offset. By so doing the high pressure shock data may be used as an independent measure of the offset of an anisotropic curve without prejudice as to the nature of the high pressure response. This then provides for an independent comparison of shock data and static data. A schematic of the method for doing this is shown in Fig. 5.

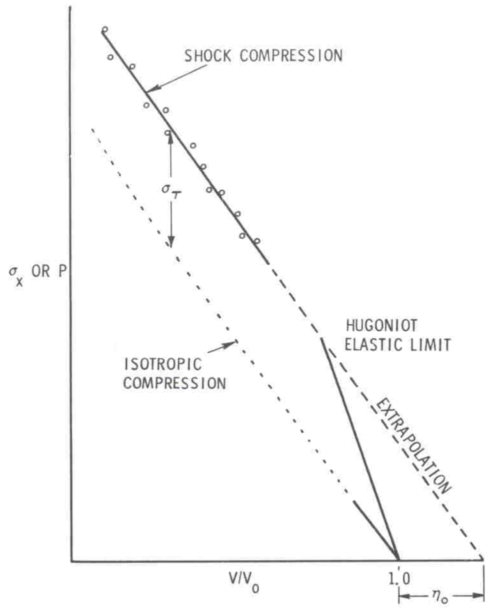


Fig. 5. Extrapolation of high pressure shock data to zero stress may be used to obtain an independent measure of the offset between shock data and the isotropic compression curve. The volume offset at zero stress is a direct measure of the volume offset and the stress offset may be computed as described in the text. The solid depicted responds as an elastic–plastic solid such that $\sigma_\tau = \sigma^*$. The extrapolation does not involve any *a priori* assumptions concerning shear strength and thus may be used as an independent measure of stress offsets.

Assuming that the shear stress offset, $\sigma_\tau$, is constant for all data above the HEL it can be shown that

$$\sigma_\tau = B\eta_0 \qquad (12)$$

where $B$ is the bulk modulus, $\eta_0$ is the magnitude of the zero-stress relative volume offset measured from the extrapolation of the high pressure shock data. The bulk modulus must be known but may be determined from data at atmospheric pressure. The change in bulk modulus with pressure will not seriously affect the calculation since only the change in modulus for compressions up to a value equal to the volume offset will affect the calculation.

The zero stress relative volume offset for sapphire is determined from the shock data above the HEL with a cubic polynomial fit to the data. A cubic polynomial expansion about the initial volume, $V_0$, shows that:

$$1-\frac{V}{V_0} = -\eta_0 + A\sigma - B\sigma^2 + C\sigma^3, \qquad (13)$$

where $A = 1/B_0$ and $B_0$ is the atmospheric pressure bulk modulus. If the bulk modulus is assumed to be linear in pressure; [2]$B = (1+B_0')/2B_0^2$ and $C = (1+3B_0'+2(B_0')^2)/6B_0^3$, where $B_0'$ is the first pressure derivative of the bulk modulus. The adequacy of the cubic polynomial to represent incompressible solids like sapphire in this pressure range has been discussed by Anderson[2].

The best fit for the zero stress volume offset was judged by requiring the first linear term of the polynomial to agree with the ultrasonic measurement of compressibility. The volume offset so obtained is the initial point of a high pressure compression curve fit to the shock data with initial slope in agreement with the ultrasonic data. Iterations are performed in which a volume offset is assumed and fit to the data until agreement with the adiabatic compressibility is obtained. This technique was found to give a volume offset to three significant figures with the present data. Since the coefficient of the second term is related to the pressure derivative of the bulk modulus, $B_0'$, this value is also obtained from the polynomial fit. The values derived for the present shock data are: $\eta_0 = 0{\cdot}0118$, $B_0 = 2{\cdot}542 \times 10^3$ kbar, and $B' = 3{\cdot}69$. Because the compressions are relatively small, these values are representative of adiabatic compressions.

Using a bulk modulus value of $2{\cdot}542 \times 10^3$ kbar[36], the shear stress offset of $\sigma_\tau = 30$ kbar is obtained from equation (12); whereas, the calculated shear strength offset, $\sigma^*$ from equation (7) has values which range from 55 to 110 kbar.*

From another point of view, the relative volume offset can be used to calculate an HEL value consistent with the elastic–plastic theory and this value can be compared to the measured HEL values. Assuming constant values for bulk modulus, $B$, and the appropriate one-dimensional strain elastic constant, $C$, up to compressions to the HEL, it can be shown that if the solid responds as an elastic-plastic solid that:

$$\sigma_\eta = B\eta_0(1-B/C)^{-1} \qquad (14)$$

where $\sigma_\eta$ is the computed HEL consistent with the observed volume offset and the elastic–plastic model.

The calculated HEL from equation (13) is 60 kbar. This value should be compared to the measured HEL values which range from 120 to 210 kbar. The discrepancy between the HEL predicted from the elastic–plastic theory and the measured values indicates that the elastic–plastic model will not describe the compression of sapphire.

### (f) *Summary of shear strength observations*

In summary, it is clear that there are three

*All calculations involving elastic constants in this paper use the data of Gieske and Barsch, Ref. [36]. Their paper also contains an excellent summary of other elastic constant data on $Al_2O_3$. For the single crystal $\tau^*$ was calculated from: $\tau^* = \frac{1}{2}(1-C_{xz}/C_{xx})\sigma_H$, where $x$ is the shock propagation direction and $z$ is the lateral direction and the $C$'s are the elastic stiffness constants.